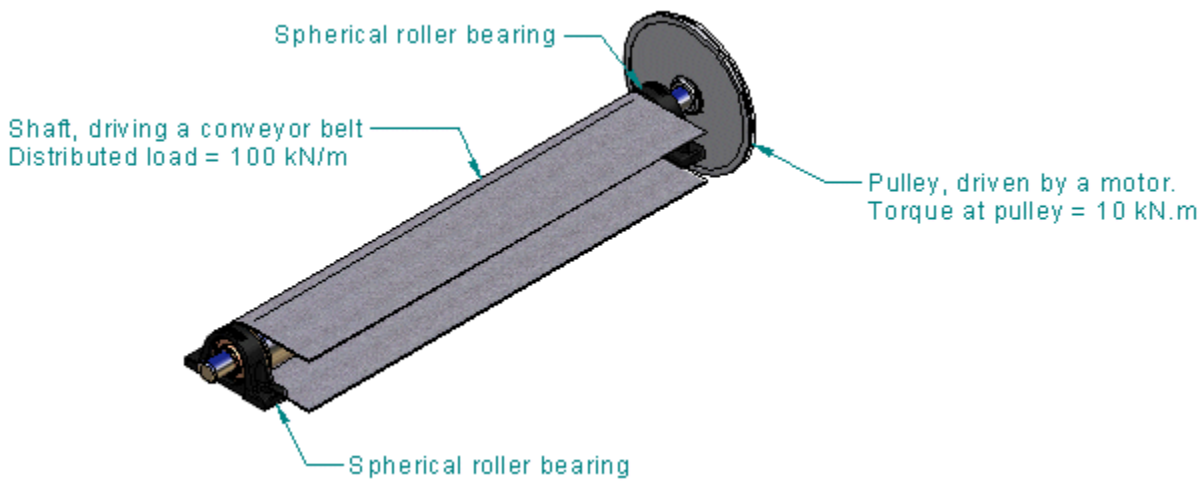


Motivated Design & Analysis, Ltd.
MathCAD Stress Analysis
Analysis Title: FEA Brainteaser # 2
Analyst: Greg
Checked by:
Rev N/C, Updated 08 May, 2006

N := newton
kN := $10^3 \cdot \text{newton}$
Tonne := $10^3 \cdot \text{kg}$
MPa := $10^6 \cdot \text{Pa}$
GPa := $10^9 \cdot \text{Pa}$
rev := 360 \cdot \text{deg}

Problem #2: Harder System, Round Bar in Bending and Torsion, evenly distributed Load, simply supported:

Outline: A shaft, pulley and conveyor belt assembly is shown in the figure below. Find the maximum stress in the shaft under the given operating conditions:



Dimensions (in mm):

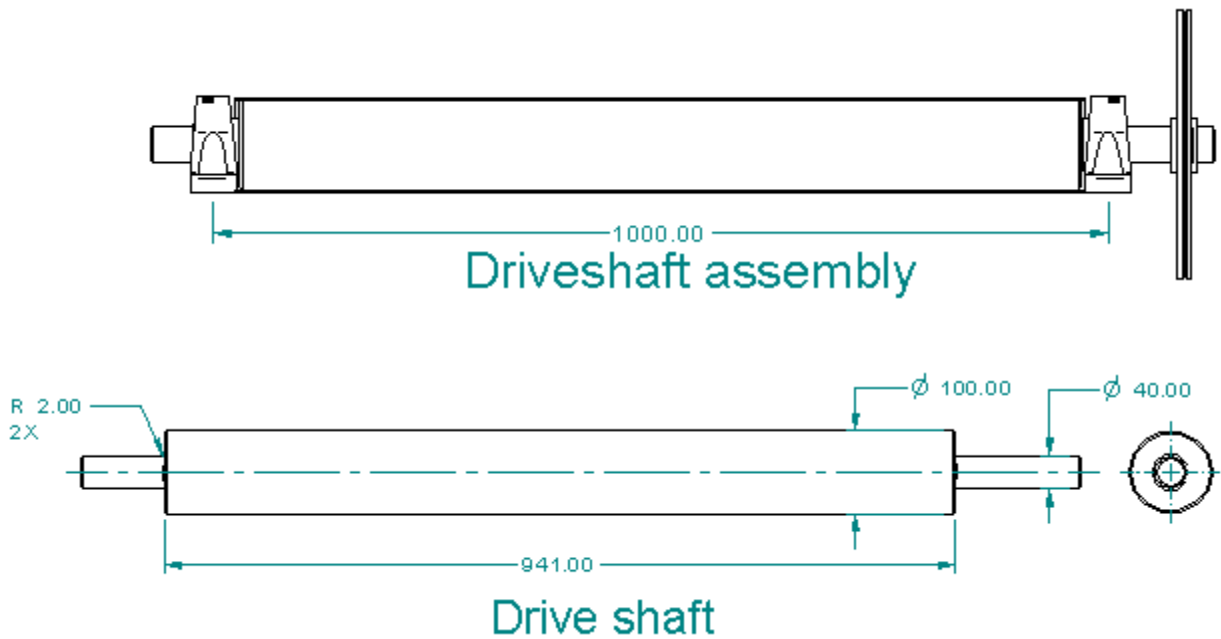


Figure 1: Drive shaft problem outline and geometry

Distributed load

$$w_{\text{dist}} := 100 \cdot \frac{\text{kN}}{\text{m}}$$

$$L := 1000 \cdot \text{mm}$$

$$L_{\text{dist}} := 941 \cdot \text{mm}$$

Applied force:

$$W := w_{\text{dist}} \cdot L_{\text{dist}}$$

$$W = 94.1 \cdot \text{kN}$$

Applied torque

$$T := 10 \cdot \text{kN} \cdot \text{m}$$

Section dimensions

$$D := 100 \cdot \text{mm}$$

$$\text{radius} := 2 \cdot \text{mm}$$

$$d := 40 \cdot \text{mm}$$

Look at stresses at the centre of the shaft

$$r_{\text{c}} := \frac{D}{2}$$

$$J_{\text{c}} := \frac{1}{2} \cdot \pi \cdot r_{\text{c}}^4$$

$$J_{\text{c}} = 1 \cdot 10^7 \cdot \text{mm}^4$$

$$I_{\text{RD.c}} := \frac{1}{4} \cdot \pi \cdot r_{\text{c}}^4$$

$$I_{\text{RD.c}} = 4.91 \cdot 10^6 \cdot \text{mm}^4$$

Bending moment for a distributed load. Note that this formula is slightly inaccurate as the distributed load is not applied right up to the bearings, but is conservative.

$$M_{\text{RD.c}} := \frac{w_{\text{dist}} \cdot L^2}{8}$$

$$M_{\text{RD.c}} = 12500 \cdot \text{N} \cdot \text{m}$$

Section modulus

$$S_{\text{RD.c}} := \frac{I_{\text{RD.c}}}{r_{\text{c}}}$$

$$S_{\text{RD.c}} = 98175 \cdot \text{mm}^3$$

Stress due to bending at the centre of the shaft is:

$$\sigma_{\text{RD.c}} := \frac{M_{\text{RD.c}}}{S_{\text{RD.c}}}$$

$$\sigma_{\text{RD.c}} = 127 \cdot \text{MPa}$$

Shear stress due to torsion at the centre of the shaft is:

$$\tau_{\text{c}} := \frac{T \cdot r_{\text{c}}}{J_{\text{c}}}$$

$$\tau_{\text{c}} = 50.9 \cdot \text{MPa}$$

Note: this shear stress calculation is a conservative assumption, as the torque will actually decrease linearly across the shaft from the driven end...however, this should correlate to the FEA results.

The equivalent stress, since we are going to compare Von Mises stresses, is

$$\sigma_{\text{eq.c}} := \sqrt{\sigma_{\text{RD.c}}^2 + 3 \cdot \tau_{\text{c}}^2} \quad \sigma_{\text{eq.c}} = 155 \cdot \text{MPa}$$

Look at stresses at the shaft shoulder

$$r_s := \frac{d}{2}$$

$$r_s = 20 \cdot \text{mm}$$

$$J_s := \frac{1}{2} \cdot \pi \cdot r_s^4$$

$$I_{RD,s} := \frac{1}{4} \cdot \pi \cdot r_s^4$$

$$I_{RD,s} = 1.26 \cdot 10^5 \cdot \text{mm}^4$$

$$J_s = 3 \cdot 10^5 \cdot \text{mm}^4$$

The distance from the bearing reaction to the shaft shoulder is:

$$x_s := \frac{L - L_{\text{dist}}}{2}$$

$$x_s = 29 \cdot \text{mm}$$

Bending moment at the shaft shoulder for a distributed load. Again, this load is slightly conservative.

$$M_{RD,s} := \frac{W_{\text{dist}}}{2} \cdot x_s \cdot (L - x_s)$$

$$M_{RD,s} = 1431 \cdot \text{N} \cdot \text{m}$$

Section modulus

$$S_{RD,s} := \frac{I_{RD,s}}{r_s}$$

$$S_{RD,s} = 6283 \cdot \text{mm}^3$$

Stress due to bending at the shaft shoulder is:

$$\sigma_{RD,s} := \frac{M_{RD,s}}{S_{RD,s}}$$

$$\sigma_{RD,s} = 228 \cdot \text{MPa}$$

Shear stress due to torsion at the shaft shoulder is:

$$\tau_s := \frac{T \cdot r_s}{J_s}$$

$$\tau_s = 795.8 \cdot \text{MPa}$$

Wow! Very high...

The torsion kills this shaft!

Account for the stress concentration at the shoulder radius. From Shigley, the appropriate factors from stress concentration charts are:

$$\frac{\text{radius}}{d} = 0.05$$

$$\frac{D}{d} = 2.5$$

For normal stress: $K_t := 2.2$ for a shaft in bending

For shear stress: $K_{ts} := 1.8$ for a shaft in torsion

$$\sigma_{\text{eq},s} := \sqrt{(K_t \cdot \sigma_{RD,s})^2 + 3 \cdot (K_{ts} \cdot \tau_s)^2}$$

$$\sigma_{\text{eq},s} = 2531 \cdot \text{MPa}$$

(answer)

Obviously this design is very overstressed, which wouldn't be obvious from the bearing selection (the static load capacity of a spherical roller bearing for a 40mm diameter shaft is 140kN, which is well above the bearing load required). Good thing we did an analysis!

Free CAD embedded FEA Results

1. Approach: Again, check to see if symmetry can be used. The torque load is independent of position, and the distributed load is applied at the centre of the shaft, so symmetry kind of applies. So the model was again cut in half.

2. Assumptions: The same assumption of fixing the face of symmetry as in Problem # 1 still applies.

3. Applied loads: A free-body diagram of the shaft shows the two applied loads (distributed load from conveyor belt, and driving torque from the pulley) and two reaction forces from the bearings. By inspection, the reaction forces are half the applied force.

$$R_{\text{brg.1}} := \frac{W}{2}$$

$$R_{\text{brg.1}} = 47.05 \cdot \text{kN}$$

$$F_{\text{dist.sym}} := w_{\text{dist}} \cdot \frac{L_{\text{dist}}}{2}$$

$$F_{\text{dist.sym}} = 47.05 \cdot \text{kN}$$

Apply the distributed load as a pressure. Use a unit area of $941/2\text{mm} \times 1\text{mm}$. This gives an equivalent pressure of:

$$A_{\text{dist.load}} := \frac{L_{\text{dist}}}{2} \cdot 1 \cdot \text{mm}$$

$$A_{\text{dist.load}} = 471 \cdot \text{mm}^2$$

$$P_{\text{dist.load}} := \frac{F_{\text{dist.sym}}}{A_{\text{dist.load}}}$$

$$P_{\text{dist.load}} = 100 \cdot \text{MPa}$$

The distributed load will be applied at the centre portion of the shaft (although applying it at the top edge is technically more correct). Note that this could also have been applied directly as a force.

The torque load must be applied in such a way as to not induce any additional bending in the shaft, and so should be applied as a couple. There are several ways to achieve this using limited capability software, as the only load options are forces and pressures. We modelled in a pulley and applied appropriate forces to the top and bottom edges.

pulley diam := 400 · mm (choose a pulley diameter and calculate the required force)

$$F_{\text{pulley}} := \frac{T}{\text{pulley diam}}$$

$$F_{\text{pulley}} = 25 \cdot \text{kN}$$

4. Initial results: the first run was completed with the default mesh size, with the bending and twisting loads applied independantly to verify that they result in the expected deformations (Figures 2 and 3).

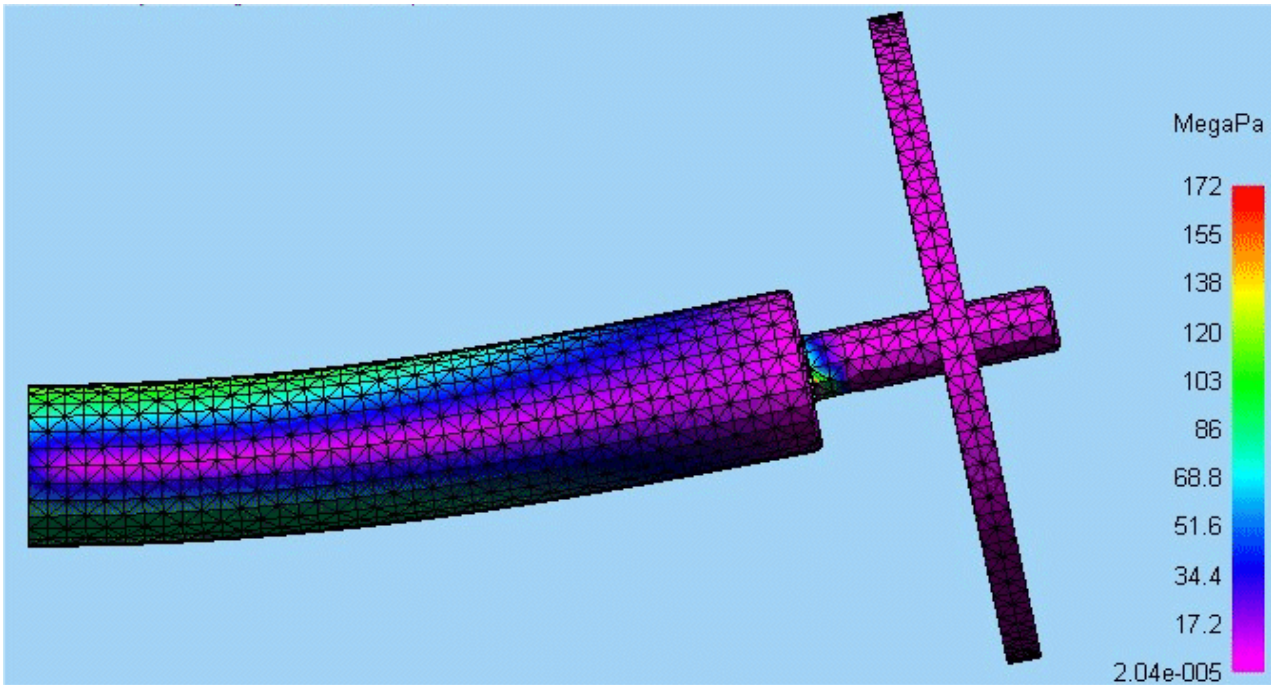


Figure 2: Drive shaft analysis, bending loads only, Von Mises stress (MPa)

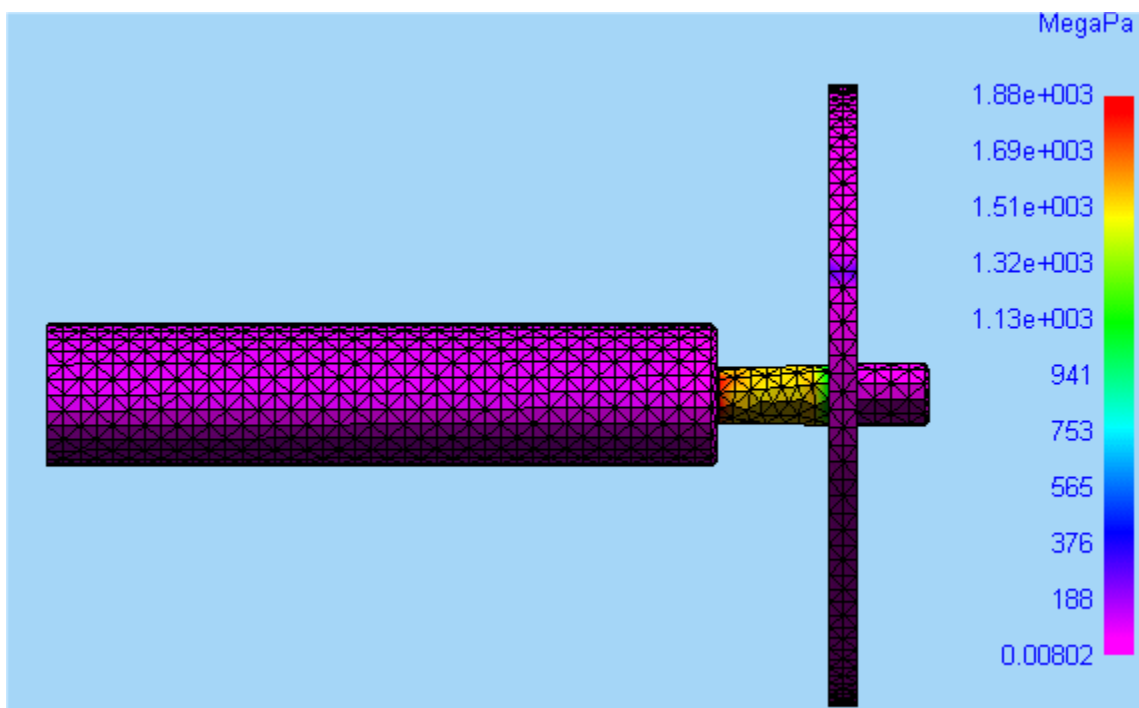
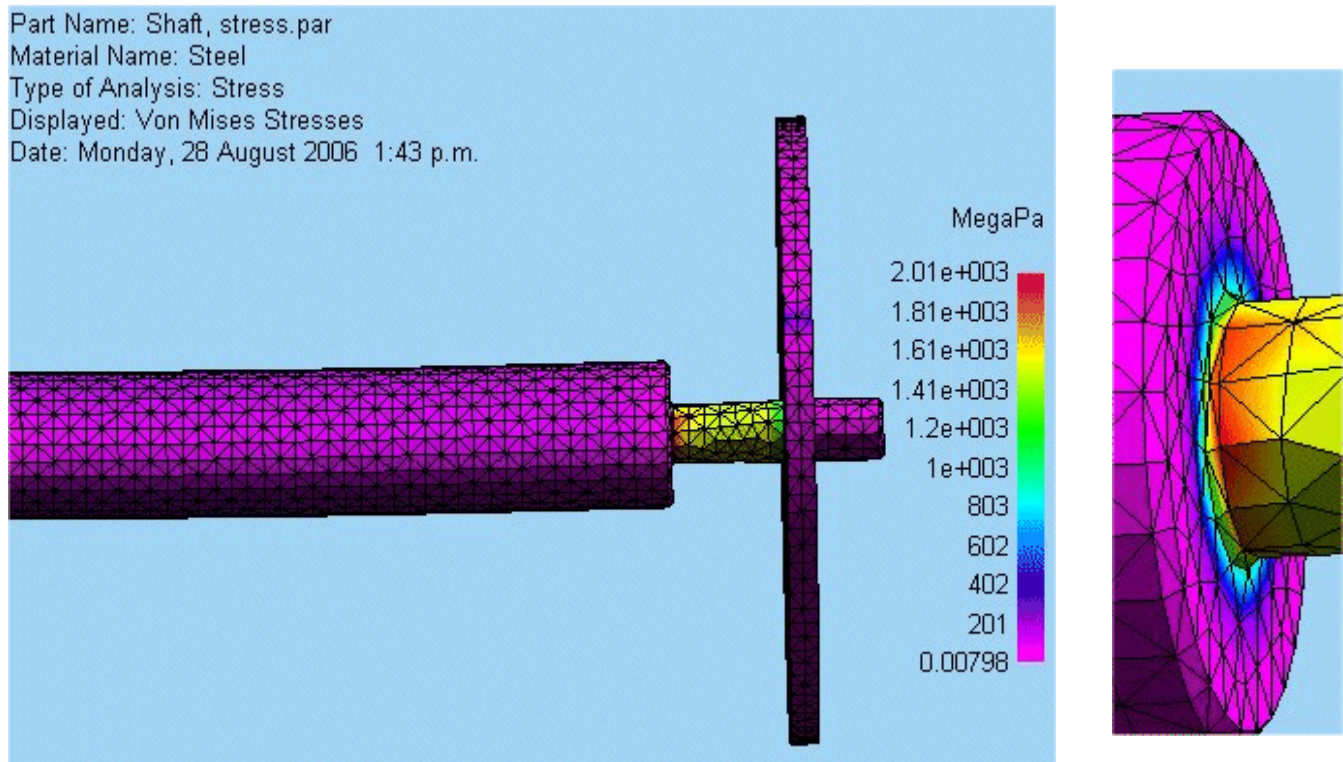


Figure 3: Drive shaft analysis, torsion load only, Von Mises stress (MPa)

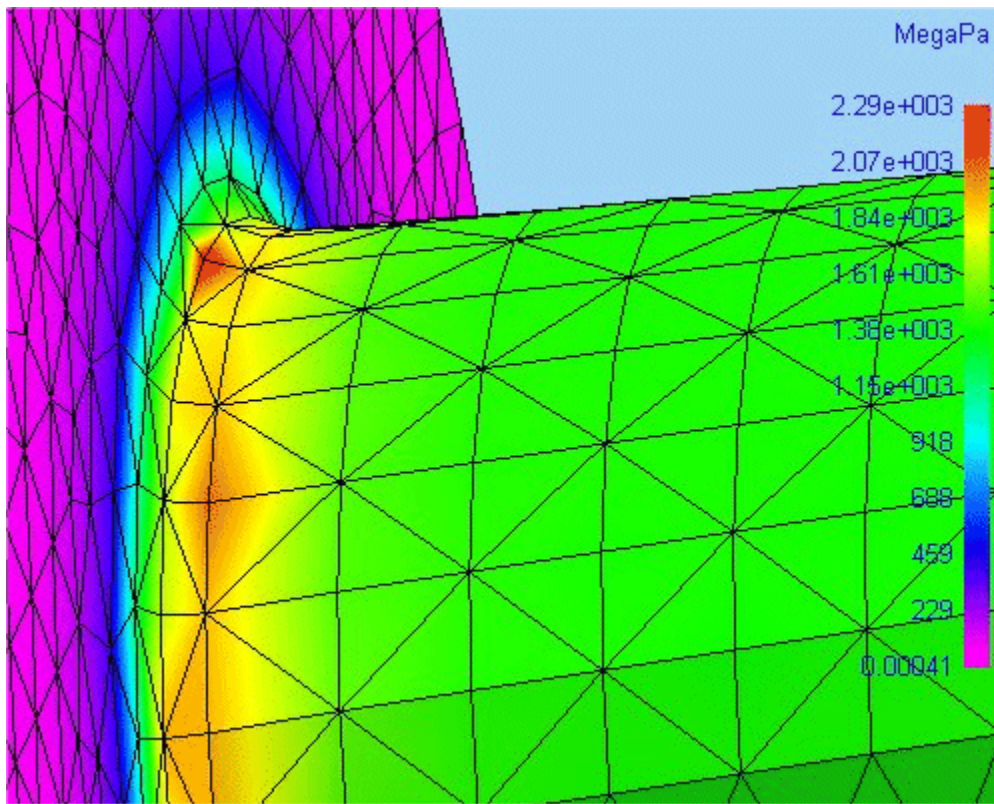
Note that high stresses are also highlighted at the shoulder radius. However, the bending stress correlates well, at around 120MPa.

The torsional stress at the centre of the shaft is in the right order of magnitude, but limitations with the graph scale only allow the stress to be guessed at between 0 and 188MPa.

5. Final solution: combined loads.



There is only one element across the shoulder radius. Refine mesh as much as possible. Note the limitation again of mesh refinement, especially in comparing results.



The stresses are very high, well above the ultimate stress of a good steel. This magnitude of this stress is also verified by the calculations above. Check the percentage error.

$$\sigma_{\text{FEA.2}} := 2.29 \cdot 10^3 \cdot \text{MPa}$$

$$\sigma_{\text{FEA.2}} = 2.29 \cdot \text{GPa}$$

$$\text{error}_{\%.2} := \frac{\sigma_{\text{FEA.2}} - \sigma_{\text{eq.s}}}{\sigma_{\text{eq.s}}}$$

$$\text{error}_{\%.2} = -9.53 \cdot \%$$

Remember that this is a linear, static analysis, and stresses above yield will not be represented correctly. This shaft will fail, in short order.

To remedy this problem, increase the diameter of the shoulder and increase the size of the radius. To be sure, a fatigue analysis should be performed on features like this if you can't get the stresses down below an appropriate allowable stress.